Universität Stuttgart
Institut für Flugmechanik und Flugregelung

NONLINEAR SYSTEM ANALYSIS

State estimation with Sum-of-Squares programming

Master Thesis

Motivation

The general problem of estimating the state of a nonlinear system given noisy measurement data is critical for system monitoring and controller feedback stability. Recent advances in sum-of-squares programming led to the development of multiple tools in analysis of nonlinear systems without linearization. Limited work has been done to use sum-of-squares in state estimation and how does applying this tool compares to conventional state estimation techniques such as extended Kalman filter.

Problem Statement

Our attention is directed to discrete-time systems as follows,

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{f}_k(\boldsymbol{x}_k) + \boldsymbol{w}_k, \qquad k = 1, 2, \dots \\ \boldsymbol{z}_k &= \boldsymbol{h}_k(\boldsymbol{x}_k) + \boldsymbol{v}_k, \qquad k = 1, 2, \dots \end{aligned} \tag{1}$$

With $\boldsymbol{f}_k$ and $\boldsymbol{h}_k$ being nonlinear functions, and $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ being process disturbance and measurement noise, respectively. Algorithms such as the one presented in [1] leverages sum-of-squares programming and works closely to the real system dynamics without a great loss of the system dynamics. We seek to obtain a deeper insight into these algorithms and their performance compared to conventional approaches.

Tasks

- Literature review on state estimation techniques on non-linear systems [3, 4]
- Study of Sum-of-Squares programming [2, 5]
- Implementation of multiple algorithms in order to perform a comparative study
- Further study into state estimation with Sum-of-Squares programming [1]
- Optional: Propose a novel strategy

Requirements

Have or be motivated to acquire,

- Good knowledge in convex and non convex programming (previous knowledge in Sum-of-Squares programming is advantageous)
- Good understanding of Kalman filters and non-linear system theory
- Mathematical formalism and rigor

The thesis is to be written in English. Publication of results is envisaged.

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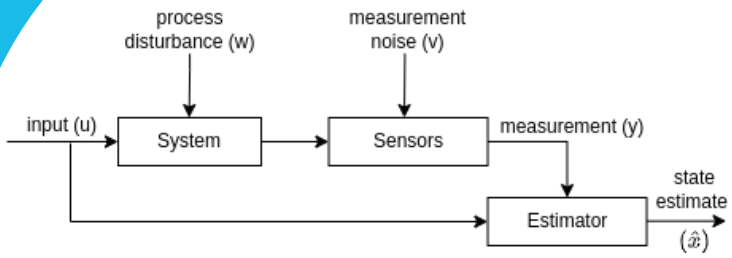

Literature

[1] G. Hexner and H. Weiss, "An extended Kalman filter with a computed mean square error bound," IEEE Conference on Decision and Control, Los Angeles, CA, USA, 2014, pp. 5008-5014.
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[2] Papachristodoulou, A., & Prajna, S. (2005). "A tutorial on sum of squares techniques for systems analysis". Proceedings of the American Control Conference, 4, 2686–2700.
https://doi.org/10.1109/acc.2005.1470374

[3] Simon, D. (2010). "Kalman filtering with state constraints: A survey of linear and nonlinear algorithms". IET Control Theory and Applications, 4(8), 1303–1318.
https://doi.org/10.1049/iet-cta.2009.0032

[4] H.W. Sorenson, "On the development of practical nonlinear filters", Information Sciences, Volume 7, 1974, Pages 253-270, ISSN 0020-0255.
https://doi.org/10.1016/0020-0255(74)90017-6

[5] W. Tan, "Nonlinear Control Analysis and Synthesis using Sum-of-Squares programming", Ph.D. dissertation, University of California, Berkeley, 2006